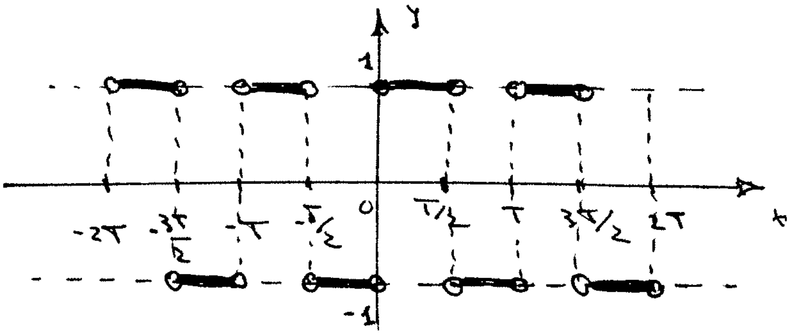
TD3, Exercice 2

1). $f(x) = \begin{cases} 1 , & 0 < x < T/2 \\ -1 , & T/2 < x < T \end{cases}$



Fonction impaire, donc développable en série de sinus:

$$S_f(x) = \sum_{n=1}^{\infty} a_n \sin n\omega x \quad , \quad \omega = 2\pi/T$$

$$a_n = \frac{2}{T}\int_0^T f(x) \sin n\omega x \, dx = \frac{4}{T}\int_0^{T/2} f(x) \sin n\omega x \, dx =$$

$$= \frac{4}{T}\int_0^{T/2} 1 \cdot \sin n\omega x \, dx = \frac{4}{T} \left. \frac{-\cos n\omega x}{n\omega} \right|_0^{T/2} =$$

$$= \frac{4}{n\omega T}\left(-\cos n\omega T/2 + 1\right) = \frac{4}{2\pi n}\left(1 - \cos \pi n\right) = \frac{2}{\pi n}\left(1-(-1)^n\right) =$$

$$= \begin{cases} 0 , & n = 2k \\ \dfrac{4}{\pi(2k+1)} , & n = 2k+1 \end{cases}$$

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$$S_f(x) = \sum_{k=0}^{\infty} \frac{4}{\pi(2k+1)} \sin (2k+1)\omega t$$

2). $f(x) = \begin{cases} 1 , & -\tau/2 < x < \tau/2 \\ 0 & \tau/2 < x < T - \tau/2 \end{cases}$