$$a_{n\neq 0} = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos n\omega x\,dx = \frac{4}{T}\int_{0}^{T/2} f(x)\cos n\omega x\,dx =$$

$$= \frac{4}{T}\int_{0}^{T/2} x\cos n\omega x\,dx = \frac{4}{T}\int_{0}^{T/2} x\, d\left(\frac{\sin n\omega x}{n\omega}\right) =$$

$$= \frac{4}{T}\left\{ x\,\frac{\sin n\omega x}{n\omega}\Big|_{0}^{T/2} - \int_{0}^{T/2}\frac{\sin n\omega x}{n\omega}\,dx\right\} =$$

$$= \frac{4}{T}\left\{ \frac{T}{2}\,\frac{\cancelto{0}{\sin n\frac{2\pi}{T}\cdot\frac{T}{2}}}{n\omega} + \frac{\cos n\omega x}{(n\omega)^2}\Big|_{0}^{T/2}\right\} =$$

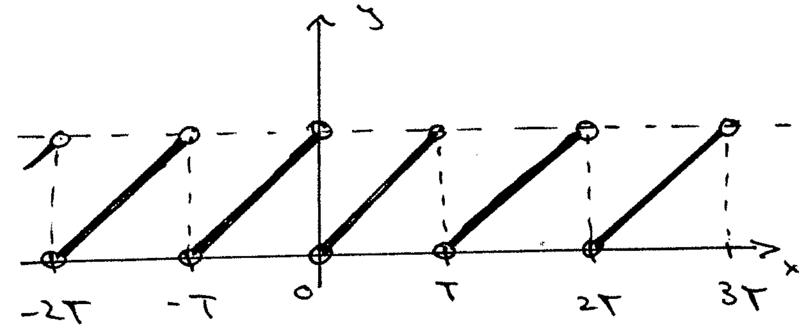
$$= \frac{4}{T}\,\frac{\cos\pi n - 1}{(n\omega)^2} = \frac{\cancel{4}\,T^2}{T\cdot n^2\cdot\cancel{4}\pi^2}\left[(-1)^n - 1\right] =$$

$$= \frac{T}{\pi^2 n^2}\left[(-1)^n - 1\right] = \begin{cases} 0, & n = 2k \\ -\dfrac{2T}{\pi^2 n^2}, & n = 2k+1 \end{cases}$$

Donc

$$f(x) = \frac{T}{4} - \frac{2T}{\pi^2}\sum_{k=0}^{\infty}\frac{\cos 2\pi(2k+1)\frac{x}{T}}{(2k+1)^2}$$

4). $f(x) = x$ pour $x \in (0, T)$



Fonction ni paire ni impaire $\Rightarrow$ sont présents sin et cos:

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n\cos n\omega x + b_n\sin n\omega x$$

où:

$$a_0 = \frac{1}{T}\int_{0}^{T} f(x)\,dx = \frac{1}{T}\int_{0}^{T} x\,dx = \frac{1}{T}\,\frac{x^2}{2}\Big|_{0}^{T} = \frac{T}{2}$$

$$a_{n\neq 0} = \frac{2}{T}\int_{0}^{T} f(x)\cos n\omega x\,dx = \frac{2}{T}\int_{0}^{T} x\cos n\omega x\,dx =$$

$$= \frac{2}{T}\int_{0}^{T} x\, d\left(\frac{\sin n\omega x}{n\omega}\right) = \frac{2}{T}\left(\cancelto{0}{x\,\frac{\sin n\omega x}{n\omega}\Big|_{0}^{T}} - \int_{0}^{T}\frac{\sin n\omega x}{n\omega}\,dx\right) =$$

$$= \frac{2}{T}\,\frac{\cos n\omega x}{(n\omega)^2}\Big|_{0}^{T} = \frac{2}{T}\,\frac{\cos 2\pi n - 1}{(n\omega)^2} = 0$$