Fonction paire → développable en série de cosinus :

$$S_f(x) = \sum_{n=0}^{\infty} a_n \cos n\omega x$$

$$a_0 = \frac{1}{T}\int_0^T f(x)\,dx = \frac{1}{T}\int_{-\delta/2}^{T-\delta/2} f(x)\,dx = \frac{1}{T}\int_{-\tau/2}^{\delta/2} 1\,dx = \frac{\tau}{T}$$

$$a_{n\neq 0} = \frac{2}{T}\int_0^T f(x)\cos n\omega x\,dx = \frac{2}{T}\int_{-\delta/2}^{T-\delta/2} f(x)\cos n\omega x\,dx =$$

$$= \frac{2}{T}\int_{-\delta/2}^{\delta/2} \cos n\omega x\,dx = \frac{4}{T}\int_0^{\delta/2}\cos n\omega x\,dx =$$

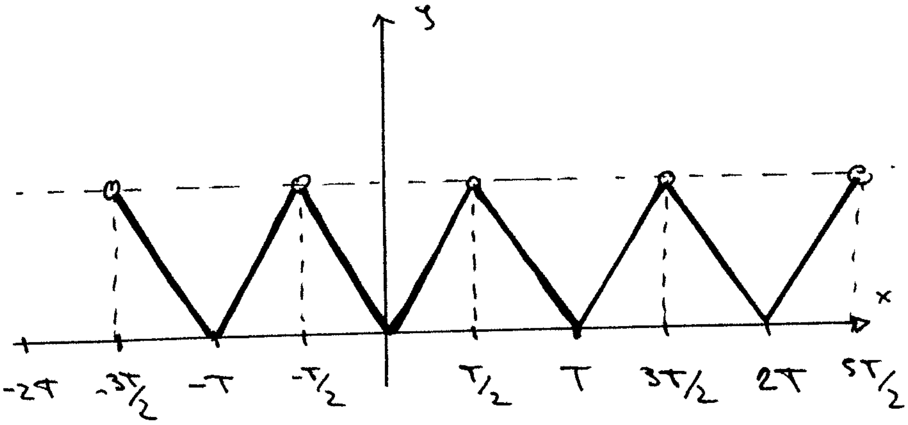
$$= \frac{4}{T}\left.\frac{\sin n\omega x}{n\omega}\right|_0^{\delta/2} = \frac{4}{T}\frac{\sin n\omega\,\delta/2}{n\omega} =$$

$$= \frac{4}{n\cdot 2\pi}\sin n\cdot\frac{2\pi}{T}\frac{\delta}{2} = \frac{2}{\pi n}\sin \pi n\frac{\delta}{T}$$

Donc

$$S_f(x) = \frac{\delta}{T} + \sum_{n=1}^{\infty}\frac{2}{\pi n}\sin\pi n\frac{\delta}{T}\cos 2\pi n\frac{x}{T}$$

3). $f(x) = |x|$ pour $x \in (-T/2, T/2)$



$f(x)$ paire ⇒ développable en série de cosinus

$$f(x) = \sum_{n=0}^{\infty} a_n \cos n\omega x$$

où

$$a_0 = \frac{1}{T}\int_0^T f(x)\,dx = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\,dx = \frac{2}{T}\int_0^{T/2} f(x)\,dx =$$

$$= \frac{2}{T}\int_0^{T/2} x\,dx = \frac{2}{T}\left.\frac{x^2}{2}\right|_0^{T/2} = \frac{2}{T}\cdot\frac{T^2}{8} = \frac{T}{4}$$