$$b_n = \frac{2}{T}\int_0^T x \sin n\omega x\,dx = \frac{2}{T}\int_0^T x\, d\left(-\frac{\cos n\omega x}{n\omega}\right) =$$

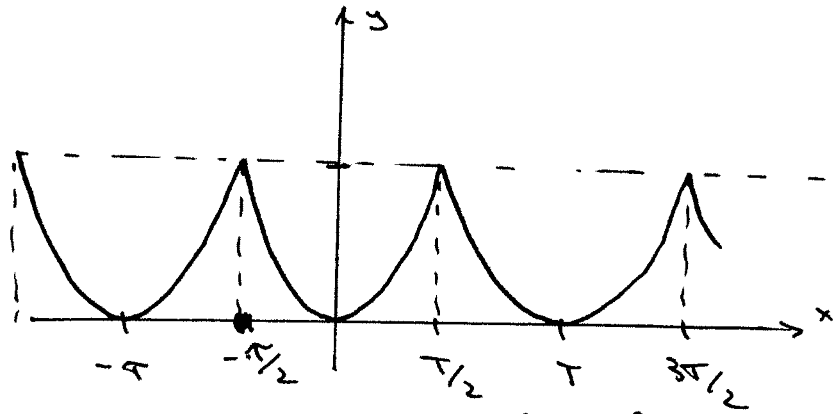
$$= \frac{2}{T}\left\{ -x\frac{\cos n\omega x}{n\omega}\Big|_0^T + \int_0^T \frac{\cos n\omega x}{n\omega}\,dx\right\} =$$

$$= \frac{2}{T}\left\{-\frac{T}{n\omega} + \cancelto{0}{\frac{\sin n\omega x}{(n\omega)^2}}\Big|_0^T\right\} = -\frac{2}{n\omega} = -\frac{T}{\pi n}$$

Donc

$$f(x) = \frac{T}{2} - \sum_{n=1}^{\infty} \frac{T}{\pi n}\sin 2\pi n x/T .$$

5). $f(x)=x^2$ pour $x\in(-T/2,T/2)$.



fonction paire $\Longrightarrow$ développable en série de cosinus

$$f(x)=\sum_{n=0}^{\infty} a_n\cos n\omega x ,$$

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} f(x)\,dx = \frac{1}{T}\int_{-T/2}^{T/2}x^2\,dx = \frac{1}{T}\, \frac{x^3}{3}\Big|_{-T/2}^{T/2} =$$

$$= \frac{1}{T}\cdot\frac{2T^3/8}{3} = \frac{T^2}{12}$$

$$a_{n\neq0} = \frac{2}{T}\int_{-T/2}^{T/2} f(x)\cos n\omega x\,dx = \frac{4}{T}\int_0^{T/2}x^2\cos n\omega x\,dx=$$

$$= \frac{4}{T}\int_0^{T/2}x^2\, d\left(\frac{\sin n\omega x}{n\omega}\right) = \frac{4}{T}\left\{x^2\cancelto{0}{\frac{\sin n\omega x}{n\omega}}\Big|_0^{T/2} -\right.$$

$$\left. - \int_0^{T/2}\frac{\sin n\omega x}{n\omega}\cdot 2x\,dx\right\} = -\frac{8}{n\omega T}\int_0^{T/2} x\, d\left(-\frac{\cos n\omega x}{n\omega}\right) =$$

$$= \frac{8}{n\omega T}\left\{x\frac{\cos n\omega x}{n\omega}\Big|_0^{T/2} - \int_0^{T/2}\frac{\cos n\omega x}{n\omega}\,dx\right\} =$$

$$= \frac{8}{n\omega T}\left\{\frac{T}{2}\,\frac{\cos\pi n}{n\omega} - \cancelto{0}{\frac{\sin n\omega x}{(n\omega)^2}}\Big|_0^{T/2}\right\} =$$